

328351(14)

B. E. (Third Semester) Examination, April-May 2021

(New Scheme)

(ET&T Branch)

MATHEMATICS-III

Time Allowed : Three hours

Maximum Marks : 80

Minimum Pass Marks : 28

Note : Attempt all questions. Part (a) of each question is compulsory. Attempt any two parts from (b), (c) and (d) from each unit.

Unit-I

1. (a) Define Laplace Transform.

2

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(b) Find Laplace of

(i) $L \{ t^2 e^{-at} \}$ 3

(ii) $te^{-2t} \sin 2t$ 4

(c) Evaluate : 7

$$L^{-1} \left\{ \frac{s^2}{s^4 + s^2 + 1} \right\}$$

(d) Solve : 7

$$(D^2 + 9)x = \cos 2t$$

if $x(0) = 1, x\left(\frac{\pi}{2}\right) = -1$ 7

Unit-II

2. (a) State Cauchy's theorem. 2

(b) If $f(z)$ is an analytic function of z , prove that :

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |Rf(z)|^2 = 2 |f'(z)|^2$$
 7

(c) Expand in the series the function,

[: 3]

$$f(z) = \frac{1}{(z+1)(z+3)} \text{ in the regions.}$$

(i) $1 < |z| < 3$

(ii) $|z| > 3$ 7

(d) By Integrating around a unit circle, evaluate :

$$\int_0^{2\pi} \frac{\cos 2\theta d\theta}{5 + 4 \cos \theta}$$
 7

Unit-III

3. (a) Define Positive and Negative correlation. 2

(b) Find the coefficient of correlation between the value X and Y . 7

$X : 1 \quad 3 \quad 5 \quad 7 \quad 8 \quad 10$

$Y : 8 \quad 12 \quad 15 \quad 17 \quad 18 \quad 20$

(c) Two judges in a beauty contest rank the ten competitors in the following order :

6 4 3 1 2 7 9 8 10 5

4 1 6 7 5 8 10 9 3 2

[4]

Do the two Judges appear agree in their standards? 7

- (d) The two regression equations of the variables x and y are $x = 19.13 - 0.87 y$ and $y = 11.64 - 0.50 x$

Find :

- (i) Mean of x
 (ii) Mean of y
 (iii) Correlation coefficient between x and y . 7

Unit-IV

4. (a) Write Rodrigue formula for $P_n(x)$. 2

- (b) Solve in series the equation :

$$9x(1-x) \frac{d^2y}{dx^2} - 12 \frac{dy}{dx} + 4y = 0. \quad 7$$

- (c) Show that :

(i) $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$

(ii) $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ 7

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- (d) Prove that :

$$\int_{-1}^1 P_m(x) P_n(x) dx = \begin{cases} 0 & ; m \neq n \\ \frac{2}{2n+1} & ; n = n \end{cases} \quad 7$$

Unit-V

5. (a) Write Lagrange's linear partial differential equation. 2

- (b) Solve :

$$x^2(y-z)p + y^2(z-x)q = z^2(x-y). \quad 7$$

- (c) Solve :

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6 \frac{\partial^2 z}{\partial y^2} = y \cos x \quad 7$$

- (d) Using the method of separation of variables, solve :

$$\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$$

given that $u(x, 0) = 6e^{-3x}$. 7